第22回

- 講演者 : **Giorgio Metafune** 氏 (Universit\`a del Salento)
  - 題目 : Weighted Rellich and Calder\'on-Zygmund inequalities and applications to degenerate operators
  - 日時 : 平成29年3月13日（月）15:30 – 16:30

In 1956, Rellich proved the inequalities \[\Bigl(\frac{N(N-4)}{4}\Bigr)^2 \int_{\mathbb{R}^N}|x|^{-4}|u|^2dx\leq\int_{\mathbb{R}^N}|\Delta u|^2dx \] for $N\not=2$ and for every $u\in C_c^{\infty}(\mathbb{R}^N\setminus \{0\})$. These inequalities have been then extended to L^p-norms: in 1996, Okazawa proved the validity of \[\Bigl(\frac{N}{p}-2\Bigr)^p\Bigl(\frac{N}{p'}\Bigr)^p \int_{\mathbb{R}^N}|x|^{-2p}|u|^pdx\leq \int_{\mathbb{R}^N}|\Delta u|^pdx \] for $1<p<\frac{N}{2}$ showing also the optimality of the constants.

Weighted Rellich inequalities have also been studied. In 1998, Davies and Hinz obtained for $N\geq 3$ and for $2-\frac{N}{p}<\alpha<\frac{2}{p'}$

\begin{align}\tag{1} C^p(N,p,\alpha)\int_{\mathbb{R}^N}|x|^{(\alpha-2)p}|u|^pdx\leq \int_{\mathbb{R}^N}|x|^{\alpha p}|\Delta u|^pdx\label{1} \end{align}

with the optimal constants $C^p(N,p,\alpha)=(\frac{N}{p}-2+\alpha)^p(\frac{N}{p'}-\alpha)^p$. Later Mitidieri showed that (\ref{1}) holds in the wider range $2-\frac{N}{p}<\alpha<\frac{N}{p'}$ and with the same constants.

In recent papers Ghoussoub and Moradifam and Caldiroli and Musina improved weighted Rellich inequalities for $p=2$ by giving necessary and sufficient conditions on α for the validity of (\ref{1}) and finding also the optimal constants $C^2(N,2,\alpha)$. In particular (\ref{1}) is verified for $p=2$ if and only if $\alpha\not=N/2+n$, $\alpha\not=-N/2+2-n$ for every $n\in\mathbb{N}_0$.

We show that (\ref{1}) holds if and only if $\alpha\not=N/p'+n$, $\alpha\not=-N/p+2-n$ for every $n\in \mathbb{N}_0$. Moreover, we use Rellich inequalities to find necessary and sufficient conditions for the validity of weighted Calder\'on-Zygmund estimates when $1<p<\infty$

\begin{align} \int_{\mathbb{R}^N}|x|^{\alpha p}|D^2u|^pdx\leq C\int_{\mathbb{R}^N}|x|^{\alpha p}|\Delta u|^p dx \end{align}

for $u\in C_c^{\infty}(\mathbb{R}^N\setminus \{0\})$.

We use spectral arguments reducing the inequality above to a spectral inequality for a degenerate operator, whose spectrum can be explicitly determined. Finally, we use both Rellich and Calder\'on-Zygmund inequalities to show generation properties of the operator above and to characterize its domain. Application to operators with discontinuous leading coefficients will be also given.

Most of the content of these lectures is based on joint work with Chiara Spina, Motohiro Sobajima, Noboru Okazawa.

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