# Japanese-European symposium on Symplectic Varieties and Moduli Spaces –third edition–

- Date: 27th (Mon.) 31st (Fri.), Aug., 2018.
- Venue: Morito Memorial Hall (conference room 1), Kagurazaka campus, Tokyo University of Science (TUS) at Kagurazaka 4-2-2, Shinjuku-ku, Tokyo 162-8601, JAPAN
- Access: either by JR (Japan Railway) or Tokyo Metro, within 10 minutes walk from **Iidabashi station**

## Program

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27th (Mon.)
10:00–10:30 Registration
10:30–10:40 Opening remarks: Prof. Y. Matsumoto, the president of TUS
10:40–12:10 M. Lehn (Universität Mainz)
14:00–15:00 G. Kapustka (Jagiellonian University)
15:30–16:30 A. Kanazawa (Kyoto University)
17:00–18:00 R. Laterveer (University of Strasbourg)

28th (Tue.)
9:30–11:00 S. Mukai (RIMS Kyoto)
11:30–12:30 K. Ueda (University of Tokyo)
14:30–15:30 C. Lehn (Technische Universität Chemnitz)
15:45–16:45 R. Yamagishi (Kyoto university)
17:00–18:00 G. Menet (University of Bourgogne)
18:15– Social dinner
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# 29th (Wed.)

9:30–11:00 M. Lehn (Universität Mainz)

11:30–12:30 L. Fu (Université Claude Bernard Lyon 1)

14:30–15:30 Y. Matsumoto (Tokyo University of Science)

16:00–17:00 G. Mongardi (University of Bologna)

17:30–18:30 G. Ouchi (IPMU / Max Planck)

30th (Thu.)

**9:30–11:00** S. Mukai (RIMS Kyoto)

11:30–12:30 S. Ma (Tokyo Institute of Technology)

14:30–15:30 A. Sarti (Université de Poitiers)

16:00–17:00 H. Uehara (Tokyo Metropolitan University)

17:30–18:30 P. Stellari (Università degli Studi di Milano)

31st (Fri.)

9:00–10:00 K. Hashimoto (University of Tokyo)

10:20–11:20 A. Perego (Università di Genova)

**11:40–12:40** Y. Nagai (Waseda University)

#### **Organization:**

Division of Modern Algebra and Cooperation with Engineering,

Director: H. Ito in Tokyo University of Science (TUS)

Scientific Committee: B. van Geemen (Milano), S. Kondo (Nagoya)

Organizing Committee: C. Camere (Milano), D. Matsushita (Hokkaido),

G. Mongardi (Bologna), H. Ohashi (TUS), U. Riess (Zürich)



#### Abstracts

## Lie Fu (Université Claude Bernard Lyon 1)

**Title** Finiteness results on the automorphism groups of compact hyperkahler manifolds.

Abstract A Klein automorphism of a complex manifold is by definition a holomorphic or anti-holomorphic diffeomorphism. I present some finiteness results I obtained in a recent joint work with Andrea Cattaneo concerning the Klein automorphism groups of compact hyper-kahler manifolds. We show that this group, as well as the (holomorphic) automorphism group, has only finitely many finite subgroups up to conjugacy. As an application in real algebraic geometry, we show that a compact hyperkahler manifold admits only finitely many real structures, i.e. anti-holomorphic involution, up to equivalence. If time permits, I will also answer the question of Prof. Oguiso on the finite generation of automorphism group of a compact hyperkahler manifold: we actually show that it is finitely presented. The preprint is available on arXiv:1806.03864.

## Atsushi Kanazawa (Kyoto University)

**Title** Toward Weil-Peterson geometry of Kahler moduli spaces of Calabi-Yau manifolds

Abstract The complex moduli space of a Calabi-Yau manifold has very rich structures, for example, the Weil-Petersson metric. In light of mirror symmetry, we expect that the Kahler moduli space has equivalently rich structures. In this talk, I will discuss an attempt to construct a natural Kahler metric on the Kahler moduli space by using Bridgeland stability conditions. This is joint work with Y.-W. Fan and S.-T. Yau.

# Grzegorz Kapustka (Jagellonian University Krakow)

**Title** Exceptional divisors of contractions of Ihs fourfolds

**Abstract** We study the geometry of exceptional divisors of birational contractions of ihs fourfolds. This is a joint work in progress with B. van Geemen.

## Robert Laterveer (University of Strasbourg)

Title Bloch 's conjecture for hyperkähler varieties.

**Abstract** Given a hyperkähler variety X with a finite-order non-symplectic automorphism f, a form of Bloch's conjecture describes the way f should act on the Chow groups of X. We will give an overview of this conjecture, and present several particular cases where the conjecture can be proven.

## Christian Lehn (Technische Universität Chemnitz)

Title Twistor Spaces and Global Torelli Theorem

Abstract Verbitsky's Global Torelli theorem has been one of the most important advances in the theory of holomorphic symplectic manifolds in the last years. In a joint work with Ben Bakker (University of Georgia) we prove a version of the Global Torelli theorem for singular symplectic varieties and discuss in how far the twistor construction is essential for the proof.

#### Manfred Lehn (Universität Mainz)

**Title** Formality problems for singularities of moduli of sheaves on K3-surfaces

**Abstract** Recently, Zhang and Budur proved that the dg-algebra governing deformations of polystable sheaves on a K3-surface is formal. I would like to give an introduction to the subject and the main ideas entering this results, beginning with Kaledin's formality obstruction.

#### Shouhei Ma (Tokyo Institute of Technology)

Title Universal abelian variety and Siegel modular forms

Abstract I give a correspondence between Siegel modular forms and pluricanonical forms on the universal family of abelian or Kummer varieties and its compactification. I first show that the ring of Siegel modular forms of weight divisible by g+2 is naturally isomorphic to the canonical ring of the nonsingular locus of the universal family. Then, for a certain class of compactification of the universal family, this extends to an isomorphism with the log canonical ring of the compactification. Moreover, I construct an explicit partial compactification and study the relationship with the Fourier-Jacobi development of modular forms. I also explain the K3 analogue.

## Yuya Matsumoto (Tokyo University of Science)

Title Degeneration of K3 surfaces and automorphisms

**Abstract** We prove that a K3 surface with an automorphism acting on the global 2-forms by a primitive m-th root of unity,  $m \neq 1, 2, 3, 4, 6$ , does not degenerate (assuming the existence of the so-called Kulikov models). This implies the compactness of moduli spaces of K3 surfaces equipped with such automorphisms. A key result used to prove this is the rationality of the actions of automorphisms on the graded quotients of the weight filtration of the l-adic cohomology groups of the surface.

#### Gregoire Menet (University of Bourgogne)

**Title** Betti numbers for irreducible symplectic orbifolds of dimension 4

Abstract It was recently established that the Global Torelli theorem can be extended to irreducible symplectic orbifolds. This provides an important motivation in order to classify the different possible Beauville-Bogomolov lattices that can be found in this context. In the smooth case, few things were done for this purpose. However, we can mention the work of Guan that established that the second Betti number of a Hyperkahler fourfold is contained between 3 and 23. In this talk we will show that this result of Guan can be extended to irreducible symplectic orbifolds and we will provide examples for several of the possible Betti numbers. This is a joint work in progress with Lie Fu.

## Giovanni Mongardi (University of Bologna)

Title Curve classes on irreducible holomorphic symplectic manifolds

**Abstract** In this talk, we prove the integral Hodge conjecture for one cycles on some (projective) IHS manifolds and discuss its consequences on Fano manifolds related to them. This is joint work with J.C. Ottem.

## Shigeru Mukai (RIMS Kyoto)

Title Enriques surfaces: fundamental polarizations and their mirror families

Abstract Enriques surfaces have 9 types of fundamental polarizations corresponding to the vertices of T237-diagram except for the right most one. Similarly nodal (or A1-type) Enriques surfaces have 7 types corresponding to 7 maximal sub-diagrams of T246 of ADE type. After quick review on what is known on these polarizations and on the period of Allcock type, I will move to 13 mirror families, that is, 1-dimensional families of Enriques surfaces characterized by these ADE sub-diagrams. Two mirror families corresponding to w2 and w4 appear in the classification of Enriques surfaces with finite groups by Nikulin and Kondo, and one corresponding to w6 is studied in our joint paper with H. Ohashi (2015). In some cases the Picard Fuchs equation of periods are determined.

# Yasunari Nagai (Waseda University)

Title Gulbrandsen-Halle-Hulek degeneration and Hilbert-Chow morphism

Abstract Gulbrandsen, Halle and Hulek gave a way to construct a good model of the family of Hilbert schemes associated with a semistable degeneration of surfaces without triple points. In this talk, I will explain that we can show that GHH model is isomorphic to a model explicitly constructed via  $S_n$ -equivariant toric construction by the speaker. This implies that we know the detail of singularities that arise in GHH degeneration, in particular that GHH degeneration is V-normal crossing.

## Arvid Perego (University of Genoa)

**Title** The second integral cohomology of moduli spaces of sheaves on K3 surfaces

Abstract In a recent joint work with Antonio Rapagnetta we showed that moduli spaces of ssemistable sheaves over projective K3 surfaces are examples of irreducible symplectic varieties (up to one case, namely symmetric products of K3 surfaces). As such, their second integral cohomology carries a pure weight-two Hodge structure and a compatible integral non-degenerate quadratic form. The aim of this talk is to provide a description of these structures. More precisely, we show that the second integral cohomology of a moduli space  $M_v$  is a free  $\mathbb{Z}$ -module of rank 23 which is Hodge isometric to the orthogonal of the Mukai vector v (as in the smooth case), and we compute the Fujiki constant of  $M_v$ .

## Alessandra Sarti (University of Poitiers)

Title Nikulin configurations on Kummer surfaces

Abstract A Nikulin configuration is the data of 16 disjoint smooth rational curves on a K3 surface. According to results of Nikulin this means that the K3 surface is a Kummer surface and the abelian surface in the Kummer structure is determined by the 16 curves. A classical question of Shioda is about the existence of non isomorphic Kummer structures on the same Kummer K3 surface. The question was positively answered and studied by several authors, and it was shown that the number of non–isomorphic Kummer structures is finite, but no explicit geometric construction of such structures was given. In the talk I will show how to construct explicitly non isomorphic Kummer structures on some generic Kummer K3 surfaces. This is a joint work with X. Roulleau.

#### Paolo Stellari (University of Milan)

Title Cubic fourfolds, noncommutative K3 surfaces and stability conditions

Abstract We illustrate a new method to induce stability conditions on semiorthogonal decompositions and apply it to the Kuznetsov component of the derived category of cubic fourfolds. We use this to generalize results of Addington-Thomas about cubic fourfolds and to study the rich hyperkaehler geometry associated to these hypersurfaces. This is the content of joint works with Arend Bayer, Howard Nuer, Marti Lahoz, Emanuele Macri and Alex Perry.

## Kazushi Ueda (University of Tokyo)

Title Moduli of K3 as moduli of A-infinity structures

**Abstract** We show that moduli spaces of lattice polarized K3 surfaces associated with exceptional unimodal singularities arise as moduli spaces of A-infinity structures on particular finite-dimensional graded algebras, and discuss its application to homological mirror symmetry. This is a joint work with Yanki Lekili.

# Hokuto Uehara (Tokyo Metropolitan University)

**Title** Spherical sheaves on  $D_n$ -singularities

Abstract Spherical objects in the derived category of coherent sheaves induce autoequivalences, so called twist functors. We often need knowledge of spherical objects to study the autoequivalence groups. Last year in this conference, I reported a classification of spherical sheaves on the minimal resolutions of  $D_4$ -singularities on surfaces. This time, I report my research on spherical sheaves on  $D_n$ -singularities.

## Ryo Yamagishi (Kyoto University)

**Title** The Hilbert squares of ADE surface singularities

Abstract The Hilbert schemes of points on singular symplectic surfaces are higher dimensional examples of singular symplectic varieties. I will give a characterization of such singularities for the 2-points case using fibers of symplectic resolutions. As an application, we see that the Fano varieties of lines on certain singular cubic fourfolds have the same singularities as the Hilbert squares of singular surfaces.