Seminar on Functional Analysis and Global Analysis

Organized by Kenro Furutani and Takao Kobayashi

5 $\sim$ 6/October 2009 @Noda Campus
Seminar room on the third floor, Building 4
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Topics : Analysis of spectral invariants and related operator theory

Program

5 (Monday) October

13:25- 13:30 Opening

13:30 - 14:30 Jürgen Eichhorn (Ernst-Moritz-Arndt Universität Greifswald, Germany)

- **Relative index theorems and analytic torsion for open manifolds**

14:45 - 15:30 Naoto Kumanogo (Kogakuin University)

- **An introduction to Feynman path integrals by time slicing approximation I**

15:45 - 16:30 Naoto Kumanogo

- **An introduction to Feynman path integrals by time slicing approximation II**

16:40 - 17:30 Kochi Uchiyama (Sophia University)

- **On local solutions to a radial p-elliptic equation on $(0, \infty)$**

6 (Tuesday) October

10:15 - 11:00 Yasushi Homma (Waseda University)

- **Rarita-Schwinger operator and harmonic polynomials**

11:15 - 12:15 Bernhelm Booss Bavnbek (Roskilde University, Denmark)

- **Continuous Variation of Cauchy Data Spaces:**
  – Task – Results – Open Problems –

——————— Lunch ———————

13:45 - 14:30 Atsusi Tachikawa (Tokyo University of Science)

- **Existence and regularity of a weakly harmonic map into a Finsler space with a special structure**

14:45 - 15:30 Chisato Iwasaki (Hyogo University)

- **Construction of the fundamental solution and a local index**

15:45 - 16:30 Wolfram Bauer (Ernst-Moritz-Arndt Universität Greifswald, Germany)

- **Spectral zeta function for certain classes of manifolds and its analytic continuation.**

Abstract

1. Jurgen Eichhorn

Title : Relative index theorems and analytic torsion for open manifolds

Abstract: For non-negative elliptic differential operators on closed manifolds, we have a well-established index theory and the notion of analytic torsion. On open manifolds, this becomes wrong since such operators must not have a finite dimensional kernel and cokernel and the spectrum must not be purely discrete. But there is a possibility to establish a similar theory for pairs of operators D, D', where D' is an appropriate perturbation of D. We consider pairs D^2, D'^2, where D, D' are generalized Dirac operators (e.g. Laplace operators) and D' is an appropriate perturbation of D. Doing this, there arise two canonical questions: 1. what are appropriate perturbations and 2. is $e^{-tD^2} - e^{-t\tilde{D}'^2}$ a trace class operator, where $\tilde{D}'$is a certain transformation of D'? We answer these two questions and establish in fact a very general relative index and analytic torsion theory.

2. Naoto Kumanogo

Title: An introduction to Feynman path integrals by time slicing approximation I, II

Abstract : In 1948, R. P. Feynman expressed the fundamental solution of the Schrödinger equation in an integral form, now called a path integral. The path integral was a new integral taken on over all paths connecting two points in the configulation space. Feynman explained his new integral as a limit of finite dimensional integrals, which is now called the time slicing approximation. Moreover, Feynman considered path integrals as a functional with geeral integrand, and suggested a new analysis on a path space with the path integrals and the functional derivatives. However, in 1960, R.H. Cameron proved that the measure for the path integral does not exist in a mathematically rigorous sense. Therefore, using time slicing approximation, we give a fairly general class of functionals so that Feynman path integrals with smooth functional derivatives have a mathematically rigorous meaning. More precisely, for any functional belonging

to our class, the time slicing approximation of Feynman path integral converges uniformly on compact subsets of the configuration space. Our class of functionals is closed under addition, multiplication, translation, real linear transformation and functional differentiation. Furthermore, the invariance under translation and orthogonal transformation, the interchange of the order with Riemann-Stieltjes integrals and limits, the integration by parts and the Taylor expansion formula with respect to functional differentiation, and the fundamental theorem of calculus hold in the path integrals.

3. **Koichi Uchiyma**

   Title : On local solutions to a radial p-elliptic equation on $(0, \infty)$

   Abstract : Reviewing power series description of analytic singularities of local solutions to a radial p-Laplace equation near a point σ in $(0, \infty)$, I will discuss power series construction of solutions $U(r)$ near the ends of $(0, \infty)$ to a radial p-elliptic equation $(r^{n-1}a(r)^{p/2}|U_r|^{p-2}U_r)_r + \lambda r^{n-1}\rho(r)|U|^{q-2}U = 0$ for $1 < p, q < \infty$ and nonzero real λ. A Briot-Bouquet type theorem of two varibales is used for these purposes.

4. **Yasushi Homma**

   Title : Rarita-Schwinger operator and harmonic polynomials

   Abstract : The Dirac operator is the first order differential operator acting on 1/2-spinor fields. It has many useful applications in geometry, global analysis, harmonic analysis, and physics. We can generalize the Dirac operator to the first order operators on other spinor fields with higher spin. For example, the operator on 3/2-spinor fields is called the Rarita-Schwinger operator, which is often used for the theory of realtivity. This RS operator has ellipticity and chirality. But its square is not Laplace type so that it is somewhat more difficult to get results for the RS operator than the Dirac operator. In this talk, we give a result about polynomial solutions for the RS operator on $\mathbb{R}^n$ from the viewpoint of representation theory.

5. Bernhelm Booss Bavnbek

Title : Continuous Variation of Cauchy Data Spaces:

— Task – Results – Open Problems —

Abstract. Generalizing approaches due to M. Morse and A. Floer, there have been various formulas expressing the spectral flow of a curve of selfadjoint elliptic operators, typically of Dirac operator type, on a partitioned manifold by the Maslov index of the induced curve of Cauchy data spaces along the separating hypersurface. In this talk, we

(1) sketch the history of the problem;

(2) derive the continuous variation of Cauchy data spaces from the smooth variation of the coefficients;

(3) summarize the geometric consequences; and

(4) outline the remaining challenging and unsolved problems.

The main emphasis of my talk will be on (2), which is the necessary prerequisite for all such spectral flow formulas. To do that, we assume the weak inner unique continuation property and one additional assumption out of two alternatives: *either* we shall demand the symmetry of the induced tangential operators along the separating hypersurface and admit arbitrary variation *or* we shall admit non-symmetric tangential operators but restrict to 0th order variation. The results were obtained in joint work with Matthias Lesch (Bonn) and Chaofeng Zhu (Tianjin) and are based on

- **a new construction of the invertible double of an elliptic operator on a compact manifold with smooth boundary,**
- **a related new construction of the Calderón projection, yielding a generalization and a new proof of the Cobordism Theorem, and**
- **a rather delicate analysis of sectorial projections for non-selfadjoint operators.**

6. Atsushi Tachikawa

Title : Existence and regularity of a weakly harmonic map into a Finsler space with a special structure

Abstract : In this talk we treat the existence and interior Hölder regularity of an energy minimizing weakly harmonic map from a m-dimensional Riemannian manifold (M, g) into a n-dimensional Finsler space (N, F) which is given as a modification of a Riemannian manifold (N, h).

7. Chisato Iwasaki

Title : Construction of the fundamental solution and a local index

Abstract : I will show a local version of Gauss-Bonnet-Chern theorem on Riemannian manifolds with boundary and its generalization are obtained by construction of the fundamental solution as follows: The fundamental solution for initial-boundary problem is constructed by symbolic calculus of pseudo-differential operators, if we introduce a new weight for symbols.

Also a local version of Riemann-Roch theorem on Kaehler manifolds can be obtained by the similar methods.

8. Wolfram Bauer

Title : Spectral zeta function for certain classes of manifolds and its analytic continuation.

Abstract : In this talk the spectral zeta function for intrinsic sub-Laplace operators is analyzed for certain types of manifolds including nilmanifolds and the 3-dimensional unit sphere $\mathbb{S}^3$. In all these examples the spectra can be calculated explicitly. In the case of nilmanifolds it is derived from the well-known heat kernel of the corresponding sub-Laplace operator on the nilpotent Lie group.

First, analytic methods will be explained which enable us to analyze the spectral zeta-functions, i.e. the location of poles, the residues and the derivative in zero. Then we extend our results to certain types of product manifolds. An interesting special case arises if one of the factors is the unit circle. In combination with our former results in the $\mathbb{S}^3$-case we can study a sub-Laplacian on $U(2)$, the group of 2×2-unitary matrices.