Tokyo University of Science
(former Science University of Tokyo)
Shin-jyuku (Tokyo), Noda (Chiba), Kuki (Saitama), Oshamanbe (Hokkaido)

Seminar[1] on Functional Analysis and Global Analysis

5 - 6 June 2007

@seminar room on 3rd floor, Building 4, Noda-campus
Tokyo University of Science
Yamazaki 2641, Noda Chiba Japan

organized by W. Bauer, K. Furutani and C. Iwasaki

Topics : All the lectures will be in the realm of geometric analysis, analysis on manifolds. This seminar is a part of the research project "Study of Global Analysis" in Tokyo University of Science.

key words : Analysis and geometry of Kohn-Laplacian, Hamiltonian dynamical system, sub-Riemann geometry and nilpotent analysis and geometry, geodesics and conjugate locus, heat kernel, Töplitz operator theory, Hilbert space with reproducing kernel and Berezin transformation.

Lectures :
Wolfram Bauer (Tokyo University of Science)
Der-Chen Chang (Georgetown University & National Tsing Hua University)
Lewis A. Coburn (State University of New York@Buffalo)
Kenro Furutani (Tokyo University of Science)
Chisato Iwasaki (University of Hyogo)
Kazuyoshi Kiyohara (Okayama University)
Tohru Morimoto (Nara Women's University)
Shu-ichi Ohno (Nippon Institute of Technology)
Yasuyuki Oka (Sophia University)
Kunio Yoshino (Musashi Institute of Technology)

[1]Partially supported by Tokyo University of Science, Grants-in-Aid for Scientific Research (C), No. 17540168 of JSPS represented by C. Iwasaki and Grants-in-Aid for Scientific Research (C), No. 17540202 of JSPS represented by K. Furutani.

Program

Tuesday June 5th

13:20 - 13:30 Opening

13:30 - 14:20 Tohru Morimoto

Title : Nilpotent Geo-analysis and its applications to Subriemannian Geometry I

Abstract : See attached sheet

14:30 - 15:20 Lewis A. Coburn

Title : Lipschitz estimates for Berezin's operator calculus

Abstract : See attached sheet.

15:40 - 16:20 Shu-ichi Ohno

Title : Linear relations for composition operators on H^∞

Abstract : We will characterize the compactness of linear combinations of composition operators on the Banach algebra of bounded analytic functions on the open unit disk. Futhermore we will estimate the norms and the essential norms of them. This is a joint work with Kei Ji Izuchi (Niigata University).

16:30 - 17:10 Kenro Furutani

Title : Hamilton-Jacobi equation and heat kernel

Abstract : I will discuss the structure of the heat kernel on 2-step nilpotent Lie groups. First I will explain a correspondence between the initial problem and mixed (= initial and boundary) problem for the Hamilton system of the bicharacteristic flow of the invariant Laplacian (and sub-Laplacian) on a nilpotent Lie group. The solution under the mixed condition gives a solution of a Hamilton-Jacobi equation. By Beals-Gaveau-Greiner this method was refereed to as complex Hamilton-Jacobi method and I consider it from the point of view of functional calculus. Then I will focus on the van-Vleck determinant from the point of view of functional calculus as well.

17:20 - 18:00 Chisato Iwasaki

Title : The solution of Hamilton-Jacobi equation corresponds to a degenerate operator of Grusin type (with K. Furutani)

Abstract : We will talk about a construction of a solution for the Hamilton system of the bicharacteristic flow of a polynomial which corresponds to a partial differential operator of Grusin type. This operator is an example of degenerate operator whose degeneracy is more than two.

18:30 ∼ - - - - - - Dinner at Kashiwa station area - - - - - -

Wednesday June 6th

10:10 - 11:00 Der-Chen Chang

Title : Analysis on decoupled domains in $\mathbb{C}^{n+1}$

Abstract : A domain $\Omega \subset \mathbb{C}^{n+1}$ and its boundary $\mathcal{M}$ are said to be decoupled of finite type if there exists sub-harmonic, nonharmonic polynomials $\{\mathcal{P}_j\}_{j=1,\ldots,n}$ with $\mathcal{P}_j(0) = 0$ such that

$$\Omega = \Big\{(z_1, \ldots, z_n, z_{n+1}) : \ \Im(z_{n+1}) > \sum_{j=1}^{n} \mathcal{P}_j(z_j)\Big\};$$

$$\mathcal{M} = \Big\{(z_1, \ldots, z_n, z_{n+1}) : \ \Im(z_{n+1}) = \sum_{j=1}^{n} \mathcal{P}_j(z_j)\Big\}.$$

We call the integer $m_j = 2 + \text{degree}(\Delta \mathcal{P}_j)$ the "degree" of $\mathcal{P}_j$. In this talk, we will first talk about the geometry induced by the Kohn Laplacian. Then we may discuss the sharp estimates for this operator.

11:10 - 12:00 Kazuyoshi Kiyohara

Title : Behavior of geodesics and the conjugate loci on ellipsoids

Abstract : It is well-known that the geodesic flow of any ellipsoid is completely integrable in the sense of symplectic geometry. In this talk, we would like to discuss much finer properties of the behavior of geodesics. In particular, we shall show an interesting asymptotic property of the distribution of the conjugate points along a geodesic and determine the shape of the conjugate locus of a general point. The latter result is a higher dimensional version of "the last geometric statement of Jacobi", which asserts that the first conjugate locus of a general point on any two-dimensional ellipsoid contains just four cusps.

>>>>>**- - - - - - - - - - Lunch - - - - - - - - - - - -**<<<<<

13:30 - 14:20 Tohru Morimoto

Title : Nilpotent Geo-analysis and its applications to Subriemannian Geometry II

Abstract : See attached sheet

14:30 - 15:10 Yasuyuki Oka and Kunio Yoshino

Title : Asymptotic expansions of the solutions to the heat equations with generalized functions initial value

Abstract : See attached sheet.

15:20 - 16:10 Wolfram Bauer

Title : Pluriharmonic Toeplitz operators and the Berezin transform

Abstract : For a series of weighted Bergman spaces H_h over bounded symmetric domains Ω in $\mathbb{C}^n$ and suitable algebras $\mathcal{A}$ of bounded operators on H_h containing all Toeplitz operators with bounded symbols it has been shown that compactness of $A \in \mathcal{A}$ can be characterized via the boundary vanishing condition of its Berezin transform. Such a result also is known in the unbounded setting of the Fock space $\mathcal{F}_h$ of all Gaussian square integrable entire functions on $\mathbb{C}^n$. In case of the pluriharmonic Bergman space H_{ph} over Ω the (pluriharmonic) Berezin transform B_{ph} is not one-to-one in general and even has non-compact operators in its kernel. From this point of view perhaps surprisingly we show, that via B_{ph} the same characterization of compactness holds for Toeplitz operators on the pluriharmonic Fock space $\mathcal{F}_{\mathrm{ph}}$. We give some applications of this result to the operator theory on $\mathcal{F}_{\mathrm{h}}$ and $\mathcal{F}_{\mathrm{ph}}$.

Asymptotic Expansions of the Solutions to the Heat Equations with Generalized Functions Initial Value

Kunio YOSHINO † and Yasuyuki OKA ‡

† Faculty of Knowledge Engineering, Musashi institute of technology,
‡ Department of Mathematics, Sophia University

In [5], M.Suwa characterized distributions of exponential growth, denoted by $H'(\mathbb{R}^d, K)$, by the heat kernel method. If $K = \{0\}$, then $H'(\mathbb{R}^d, K) = \mathcal{S}_1'(\mathbb{R}^d)$, where $\mathcal{S}_1'(\mathbb{R}^d)$ is the dual space of the Gel'fand-Shilov space $\mathcal{S}_1(\mathbb{R}^d)$ [2].We obtain the asymptotic expansion of the solution to the heat equation with exponential growth as follows.

Theorem 1. *Let $U(x,t) \in C^\infty(\mathbb{R}^d \times (0,\infty))$ satisfy the following conditions:*

(i) $\left(\frac{\partial}{\partial t} - \Delta\right) U(x,t) = 0$,

(ii) $\forall \epsilon > 0$, $\exists N_\epsilon \geq 0$, $\exists C_\epsilon \geq 0$ *s.t.* $|U(x,t)| \leq C_\epsilon t^{-N_\epsilon} e^{\epsilon|x|}$, $(0 < t < 1$, $x \in \mathbb{R}^d)$.

Then $U(x,t)$ has the following asymptotic expansion :

$$U(x,t) \sim \sum_{k=0}^{\infty} \frac{t^k}{k!} \Delta_x^k u \ , \ \left(u \in \mathcal{S}_1'(\mathbb{R}^d) \text{ and } u = \lim_{t\to 0} U(x,t)\right) .$$

i.e.

$$\lim_{t\to 0+} | < U(x,t) \ , \ \varphi > - \sum_{k=0}^{\frac{N}{2}} \frac{t^k}{k!} < \Delta_x^k u \ , \ \varphi > | \ t^{-\frac{N}{2}} = 0 \ ,$$

where $\Delta_x = \partial_{x_1}^2 + \cdots + \partial_{x_d}^2$.

Remark : We also obtain similar result in the case of Hyperfunctions with compact support.

R E F E R E N C E S

1. I.M.Gel'fand and G.E.Shilov *Generalized Functions, Volume 2, Space of Fundamental and Generalized Functions.* – Academy of Sciences Moscow, U.S.S.R, 1958.
2. T.Matuzawa, *A calculus approach to the hyperfunctions I.* Nagoya Math. J – 1987. – 108, p. 53–66.
3. M.Suwa, *Distributions of exponential growth with support in a proper convex cone*, Publ.Res.Inst.Math.Sci. – 2004. – 40, No 2, p. 565–603.
4. K.Yoshino, *Asymptotic expansions of heat equations with tempered distributions initial data.* preprint

Lipschitz estimates for
Berezin's operator calculus

L. A. Coburn

Abstract

F.A. Berezin introduced a general "symbol calculus" for linear operators on reproducing kernel Hilbert spaces. For the Hilbert space $H^2(\mathbf{C}^n, d\mu)$ of Gaussian square-integrable entire functions on complex n-space, $\mathbf{C}^n$, as well as for the Bergman Hilbert spaces $A^2(\Omega)$ of Euclidean volume square-integrable holomorphic functions on Ω, an arbitrary bounded domain in $\mathbf{C}^n$, I recently obtained sharp Lipschitz estimates for Berezin symbols of arbitrary bounded operators. These results and some applications are discussed.

Main references. There are four papers in the *Proceedings of the American Mathematical Society*: (1985)Mazur, Pflug and Skwarczynski; (2005) Coburn; (2006) Engliš and Zhang; (2007) Coburn.

Department of Mathematics, SUNY at Buffalo, Buffalo, New York 14260, USA

e-mail address: lcoburn@buffalo.edu

version: 5/01/2007

Nilpotent Geo-analysis and its Applications to Subriemannian Geometry

Tohru Morimoto *

In the classical theory of first order partial differential equations a fundamental rôle is played by the contact form $\omega = dz - \sum p_i dx_i$ which defines on the total space M of coordinates $(x_1, \cdots, x_n, z, p_1, \cdots, p_n)$ of independent variables, unknown function and first order derivatives a distinguished subbundle $D \subset TM$ of the tangent bundle of M of rank $2n$ by the equation $\omega = 0$. The bundle D is called a contact structure and spanned by

$$\frac{\partial}{\partial x_1} + p_1 \frac{\partial}{\partial z}, \cdots, \frac{\partial}{\partial x_n} + p_n \frac{\partial}{\partial z}, \frac{\partial}{\partial p_1}, \cdots, \frac{\partial}{\partial p_n},$$

which together with a normal vector $\frac{\partial}{\partial z}$ span the tangent space $T_a M$ at every point $a \in M$ and satisfies the same bracket relations as those of the Heisenberg Lie algebra:

$$[\frac{\partial}{\partial p_i}, \frac{\partial}{\partial x_j} + p_j \frac{\partial}{\partial z}] = \delta_{ij} \frac{\partial}{\partial z}.$$

This suggest that it is sometimes more natural to consider the tangent space at a point of the contact manifold M to be this Heisenberg Lie algebra rather than to be simply a vector space. This is one of the most typical situations that we often encounter in various geometric or analytic problems and leads to the following definition.

A filtered manifold is a differential manifold M endowed with a filtration $\{\mathfrak{f}^p\}_{p \in \mathbb{Z}}$ consisting of subbundles $\mathfrak{f}^p$ of the tangent bundle TM such that

i) $\mathfrak{f}^p \supset \mathfrak{f}^{p+1}$,

ii) $\mathfrak{f}^0 TM = 0, \quad \bigcup_{p \in \mathbb{Z}} \mathfrak{f}^p = TM$,

iii) $[\underline{\mathfrak{f}}^p, \underline{\mathfrak{f}}^q] \subset \underline{\mathfrak{f}}^{p+q} \quad$ for all $p, q \in \mathbb{Z}$,

where $\underline{\mathfrak{f}}^p$ denotes the sheaf of the germs of sections of $\mathfrak{f}^p$.

There is associated to each point x of a filtered manifold $(M, \mathfrak{f})$ a graded object

$$gr\mathfrak{f}_x = \bigoplus_{p \in \mathbb{Z}} gr_p \mathfrak{f}_x, \quad \text{with } gr_p \mathfrak{f}_x = \mathfrak{f}^p_x / \mathfrak{f}^{p+1}_x,$$

*Department of Mathematics, Nara Women's University, E-mail: morimoto@cc.nara-wu.ac.jp

which is not only a graded vector space but also has a natural Lie bracket induced from that of vector fields and proves to be a nilpotent graded Lie algebra.

Under the slogan of “nilpotent geometry”and “nilpotent analysis” we have been studying various objects and structures on filtered manifolds by letting the tangent nilpotent Lie algebras play the usual rôle of the tangent spaces, which provide us with new perspectives and methods not only in geometry but also in the theory of differential equations.

In particular, we have established a general existence theorem of local analytic solutions to a weightedly involutive analytic system of PDE's on a filtered manifold satisfying the Hörmander condition (a generalization of Cauchy -Kowalewski Theorem and Cartan Kähler Theorem) . (See T. Morimoto, Lie algebras, geometric structures and differential equations on filtered manifolds, Advanced Studies in Pure Mathematics 37, 2002, 205-252.)

On the other hand, now we recall the definition of subreimannian manifold. Let M be a differentiable manifold. We mean by a subriemannian structure on M a pair (D, g) consisting of a subbundle D of the tangent bundle TM of M and a riemannian metric g on D, that is, a rule g which assigns smoothly to each $x \in M$ an inner product g_x of the fibre D_x. A differentaible manifold equipped with a subreimannian structure is called a subriemannian manifold.

Given a subbundle D of the tangent bundle TM of a differentiable manifold M. We define sheaves $\{\mathcal{F}^p\}_{p<0}$ inductively by:

$\mathcal{F}^{-1} = \underline{D}$ (the sheaf of the germs of sections of D),

$\mathcal{F}^{p-1} = [\mathcal{F}^{-1}, \mathcal{F}^p] + \mathcal{F}^p \qquad (p < 0)$.

Then it holds that

$$[\mathcal{F}^p, \mathcal{F}^q] \subset \mathcal{F}^{p+q}.$$

If all $\mathcal{F}^p$ are vector bundles, that is, there exist subbundles $F^p \subset TM$ such that $\underline{F^p} = \mathcal{F}^p$ for all $p < 0$, we say that the subbundle D is regular and the filtered manifold $(M, \{F^p\})$ is generated by $F^{-1} = D$. Thus a subriemannian manifold (M, D, g) with D regular may be regarded as a geometric structure on a filtered manifold $(M, \{F^p\})$, the geometric structure being specified by giving a reimannian metric g on F^{-1} and subreimannian geometry is closely related to nilpotent geometry and analysis.

In this talk I will roughly explain methods of nilpotent geometry and analysis and then discuss how to apply them to subriemannian geometry. In particular, we introduce curvatures of a subriemnnin structure by constructing a Cartan connection associated with a subriemannian structure. We shall also study the automorphism group of a subriemannian manifold by examining the partial differential equations which define the infinitesimal automorphisms of the subriemannian manifold.