Hankel operators on the Segal-Bargmann space and Ψ^*-algebras by commutator methods

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For the Segal-Bargmann space $H^2(\mathbb{C}^n, \mu)$ of Gaussian square integrable entire functions on $\mathbb{C}^n$ we consider Hankel operators H_f with f in a symbol space $\mathcal{T}(\mathbb{C}^n)$. We define the Berezin transform for operators on $H^2(\mathbb{C}^n, \mu)$ and in terms of the *mean oscillation* of f we give necessary and sufficient conditions for H_f and $H_{\bar{f}}$ to be bounded, compact or to belong to the *von Neumann Schatten class* $\mathcal{S}_p$ for $1 \leq p < \infty$. We compare some aspects of these results to the case of Bergman spaces over bounded symmetric domains. There are close relations to the boundedness of the commutator $[P, M_f]$ where P denotes the Toeplitz projection and M_f is the multiplication by f. We describe how to construct spectral invariant Fréchet operator algebras $(\Psi_k^\Delta)_k$ with prescribed properties in $\mathcal{L}(H^2(\mathbb{C}^n, \mu))$ similar to the Hörmander classes $\Psi^0_{\rho,\delta}$ of zero-order pseudo-differential operators. Following general ideas by B. Gramsch we are using commutator methods and finite systems Δ of closed operators. With Δ in a class of vector fields on $\mathbb{C}^n$ this construction leads to algebras localized in cones $\mathcal{C} \subset \mathbb{C}^n$ and containing the Segal-Bargmann projection.