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Morse functions and the topology of the moduli space of positive scalar curvature metrics

It is well-known that a topology of given smooth compact manifold M completely determines whether the manifold M admits a Riemannian metric with positive scalar curvature (psc-metric). This question has been well-studied and (under some conditions) well-understood. This study is based on the fundametal results due to M. Gromov H.B.Lawson, R. Schoen, S.-T. Yau in the beginning of 80's.

However, even for such a standard manifold as the sphere S^n, it is known very little on the topological stucture of the moduli space of psc-metrics $\mathcal{M}^+(S^n)$. The goal of this talk is to present my recent results on the homotopy groups of the moduli space $\mathcal{M}^+(S^n)$.

First I would like to describe the conformal version of moduli space $\mathcal{M}(S^n)$ of all Riemannian metrics. These constructions were developed in my joint work with K. Akutagawa. It turns out that the moduli space $\mathcal{M}(S^n)$ coincides with certain classifying space which was well-understood back in 60's by means of the surgery theory and stable homotopy theory (by D. Burghelea, R. Lashof, J. Cerf and others).

Then I would like to show that the Morse function theory is closely related to the moduli space $\mathcal{M}(S^n)$: this was discovered and extensively explored by K. Igusa. In particular, the rational homotopy groups of the moduli space $\mathcal{M}(S^n)$ has been computed by means of the Morse function technique and using the analytical torsion.

Next I would like to show that, under the natural inclusion of moduli spaces $\iota : \mathcal{M}^+(S^n) \to \mathcal{M}(S^n)$, those homotopy groups are coming from the moduli space $\mathcal{M}^+(S^n)$ of psc-metrics. Here I use a particular construction (due to A. Hatcher) and some recent results by K. Igusa and S. Göthe (and J.Bismut-S.Göthe).

This lecture is intended to be accessible to all mature mathematicians (not just to experts in geometry and topology). I will do my best to provide a transparent exposition using elementary examples and pictures.